

Turbulent thermalization of the Quark-Gluon Plasma

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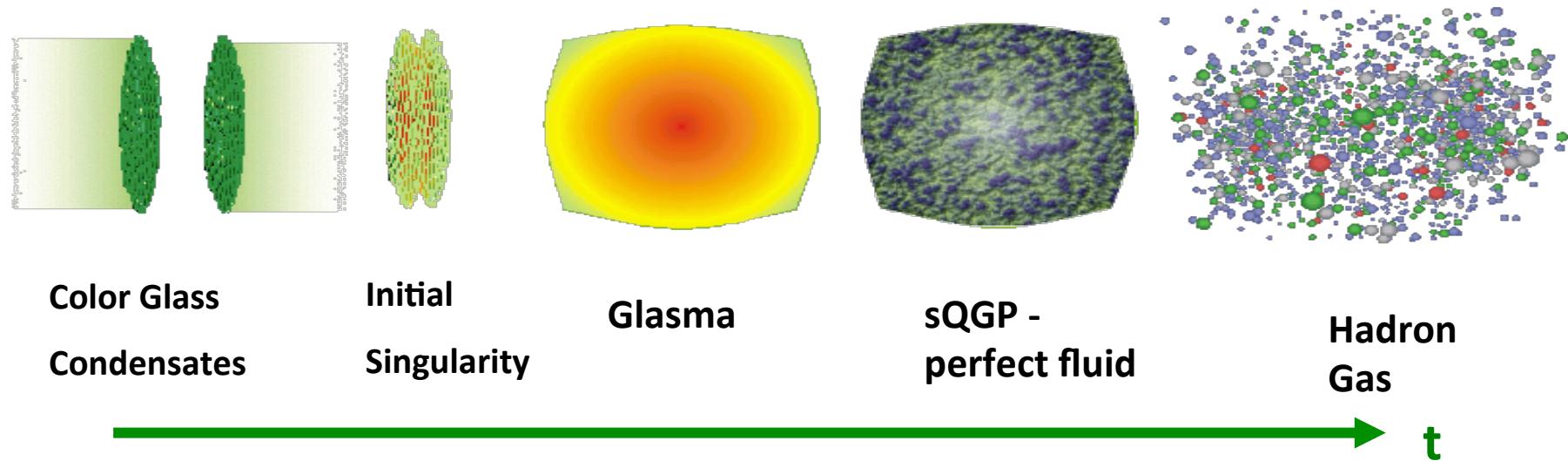
An outstanding problem in theoretical physics:

the *ab initio* understanding of the space time evolution of matter in heavy ion collisions, even in the limit $\alpha_s \ll 1$

Many effects collude to cloud (make interesting ?) our understanding of the non-equilibrium, non-Abelian dynamics:
The rapid expansion, elastic and inelastic scattering, screening, plasma instabilities, Bose-Einstein condensation, turbulence ?

Will argue definitive results can be obtained from classical-statistical dynamical simulations when gluon phase space occupancies are large ($f \gg 1$)

BNL dogma



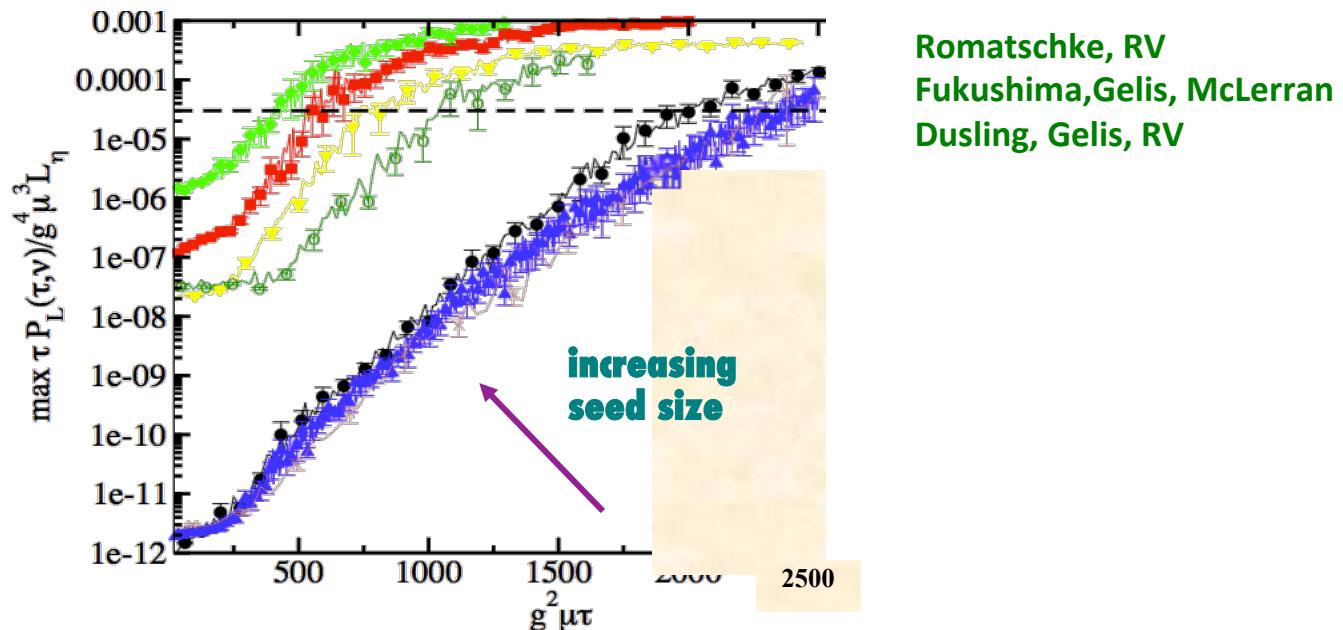
Glasma non-equilibrium regime after heavy ion collision is a Highly overpopulated system of gluons at early times:

$$n \cdot \epsilon^{3/4} \sim 1/\alpha_s^{1/4} \gg 1 \text{ for } \alpha_s \ll 1$$

Can use classical-statistical methods to simulate this system – solve longitudinally expanding 3+1-D Yang—Mills equations

Initial conditions in the Glasma

Unstable Quantum fluctuations, in analogy to preheating in inflation, play a big role, at parametrically early times $\tau = \ln^2(1/\alpha_s)/Q_s$
-- will compete with expansion at early times



Exponential growth of small initial quantum fluctuations

Initial conditions in the overpopulated QGP

Choose for the initial classical-statistic ensemble of gauge fields

$$A_\nu(\tau, \eta, x_\perp) = \sum_{\lambda} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d\nu}{2\pi} \sqrt{f_{k_\perp \nu} + \frac{1}{2}} \left[c^{(\lambda)k_\perp \nu} \xi_\mu^{(\lambda)k_\perp \nu+}(\tau) e^{ik_\perp x_\perp} e^{i\nu\eta} + c^{*(\lambda)k_\perp \nu} \xi_\mu^{(\lambda)k_\perp \nu+*}(\tau) e^{-ik_\perp x_\perp} e^{-i\nu\eta} \right]$$

with stochastic random variables

$$\begin{aligned} \langle c^{(\lambda)k_\perp \nu} c^{(\lambda')k'_\perp \nu'} \rangle &= 0, \\ \langle c^{(\lambda)k_\perp \nu} c^{*(\lambda')k'_\perp \nu'} \rangle &= (2\pi)^3 \delta^{\lambda\lambda'} \delta(k - k') \delta(\nu - \nu') \\ \langle c^{*(\lambda)k_\perp \nu} c^{*(\lambda')k'_\perp \nu'} \rangle &= 0. \end{aligned}$$

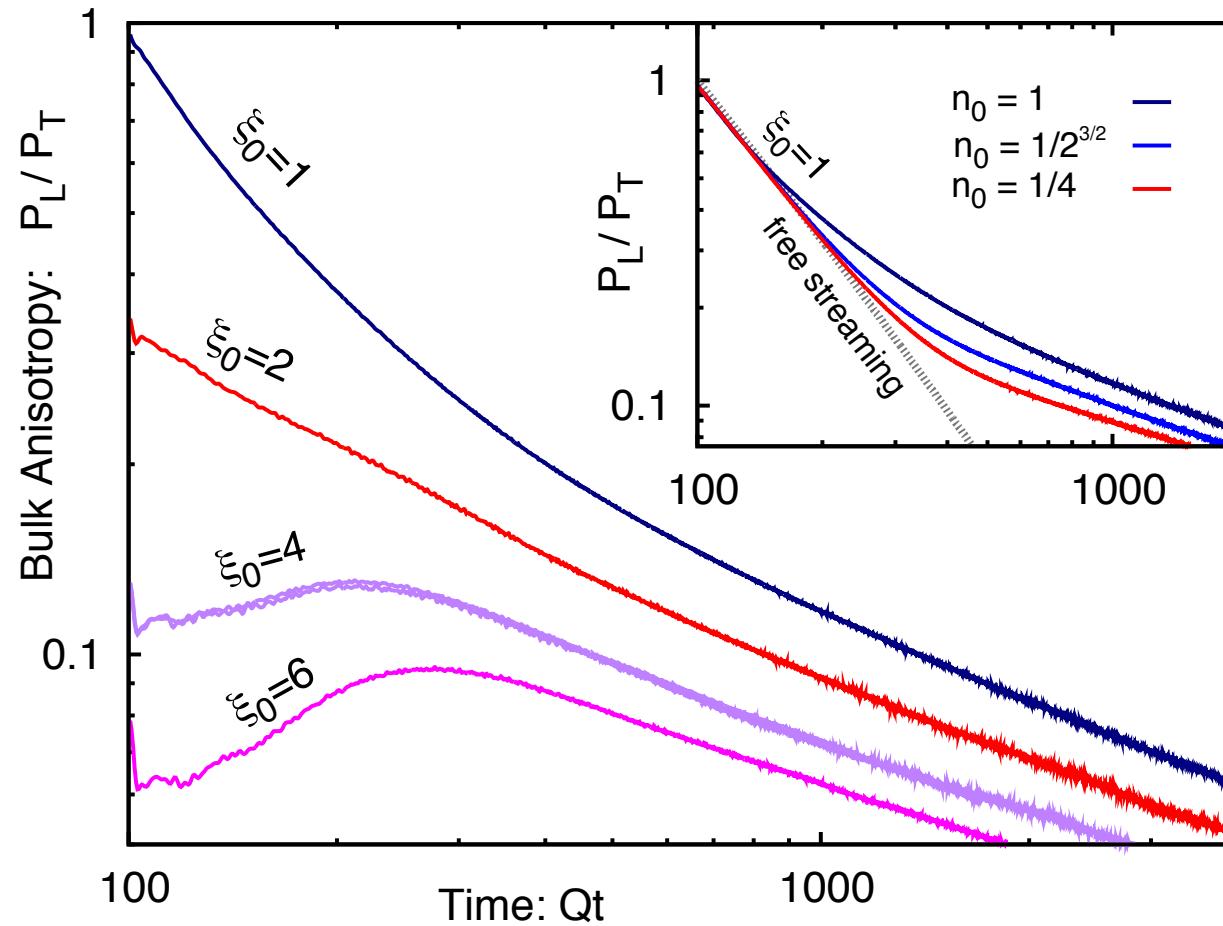
Polarization vectors ξ expressed in terms of Hankel functions in Fock-Schwinger gauge $A^\tau = 0$

$$f(p_\perp, p_z, t_0) = \frac{n_0}{\alpha_S} \Theta \left(Q - \sqrt{p_\perp^2 + (\xi_0 p_z)^2} \right)$$

With these initial conditions, solve Hamilton's eqns. for SU(2) gauge theory.
To extract gauge variant dist. function f, fix residual gauge freedom with the Coulomb condition at each readout time

$$\partial_i A_i + t^{-2} \partial_\eta A_\eta = 0$$

Results: Pressures become increasingly anisotropic



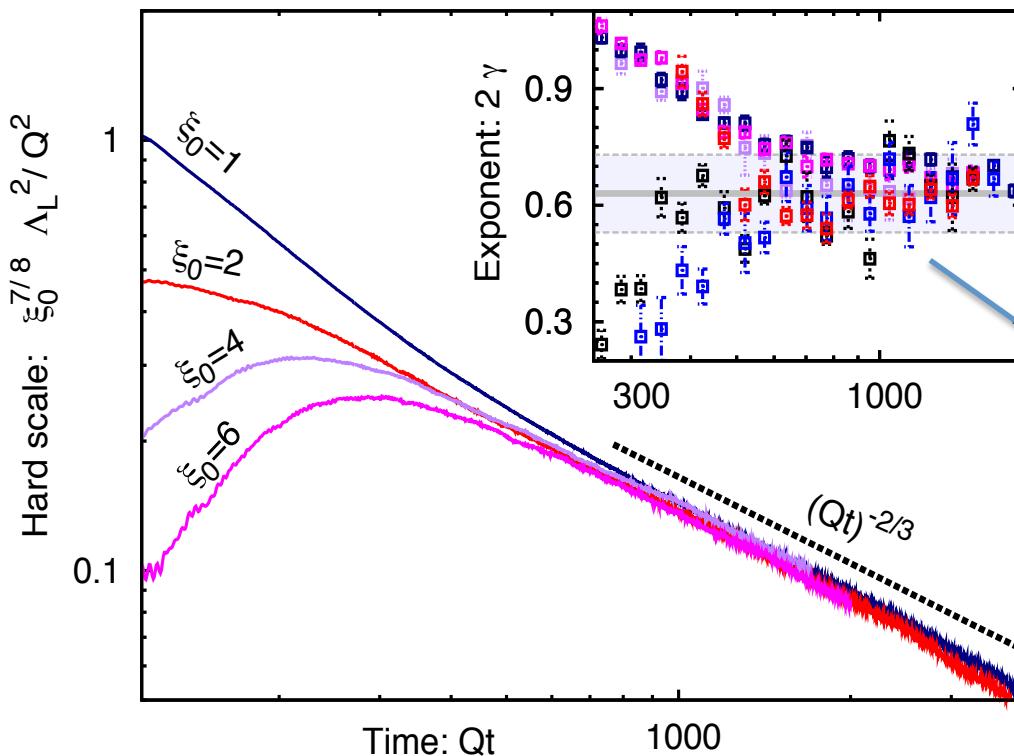
Ratios approach universal behavior independent of anisotropy parameter ξ_0 and occupancy factor n_0

Hard scales show universal scaling

Gauge invariant quantities that have simple quasi-particle interpretation for weak fields

$$\Lambda_{T,L}^2(t) \simeq \frac{\int d^2 p_T \int dp_z p_{T,z}^2 \omega_p f(p_T, p_z, t)}{\int d^2 p_T \int dp_z \omega_p f(p_T, p_z, t)}$$

$$\Lambda_L^2(t) \sim (Qt)^{-2\gamma}$$
$$\Lambda_T^2(t) \sim (Qt)^{-2\beta}$$

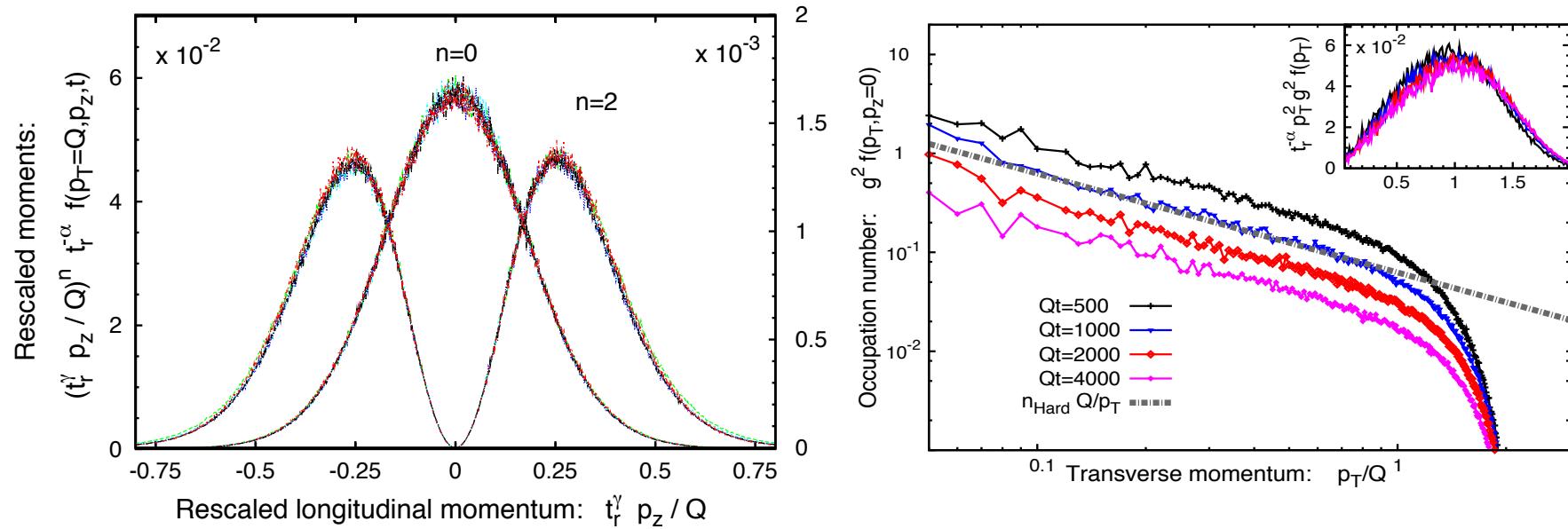


Find $\beta \approx 0$
 $\gamma \approx 1/3$

Result from large
 $(512)^2 \times 4096$ lattices

Result: universal non-thermal fixed point

Conjecture: $f(p_\perp, p_z, t) = t^\alpha f_S(t^\beta p_T, t^\gamma p_z)$



Moments of distribution extracted over range of time slices
lie on universal curves

Distribution as function of p_T displays 2-D thermal behavior

Kinetic interpretation of self-similar behavior

Follow wave turbulence kinetic picture of Zakharov, as developed by Micha & Tkachev in context of inflation

$$\left[\partial_t - \frac{p_z}{t} \partial_{p_z} \right] f(p_T, p_z, t) = C[p_T, p_z, t; f]$$

Fixed point solution satisfies

$$\Leftrightarrow C[p_T, p_z, t; f] = t^\mu C[t^\beta p_T, t^\gamma p_z; f_S]$$

$$\begin{aligned} & \alpha f_S(p_T, p_z) + \beta p_T \partial_{p_T} f_S(p_T, p_z) \\ & + (\gamma - 1) p_z \partial_{p_z} f_S(p_T, p_z) = C[p_T, p_z; f_S] \end{aligned} \quad \mu = \alpha - 1$$

If we assume that small angle elastic scattering dominates

$$C^{(\text{elast})}[p_T, p_z; f] = \hat{q} \partial_{p_z}^2 f(p_T, p_z, t)$$

$$\hat{q} \sim \alpha_S^2 \int \frac{d^2 p_T}{(2\pi)^2} \int \frac{dp_z}{2\pi} f^2(p_T, p_z, t)$$

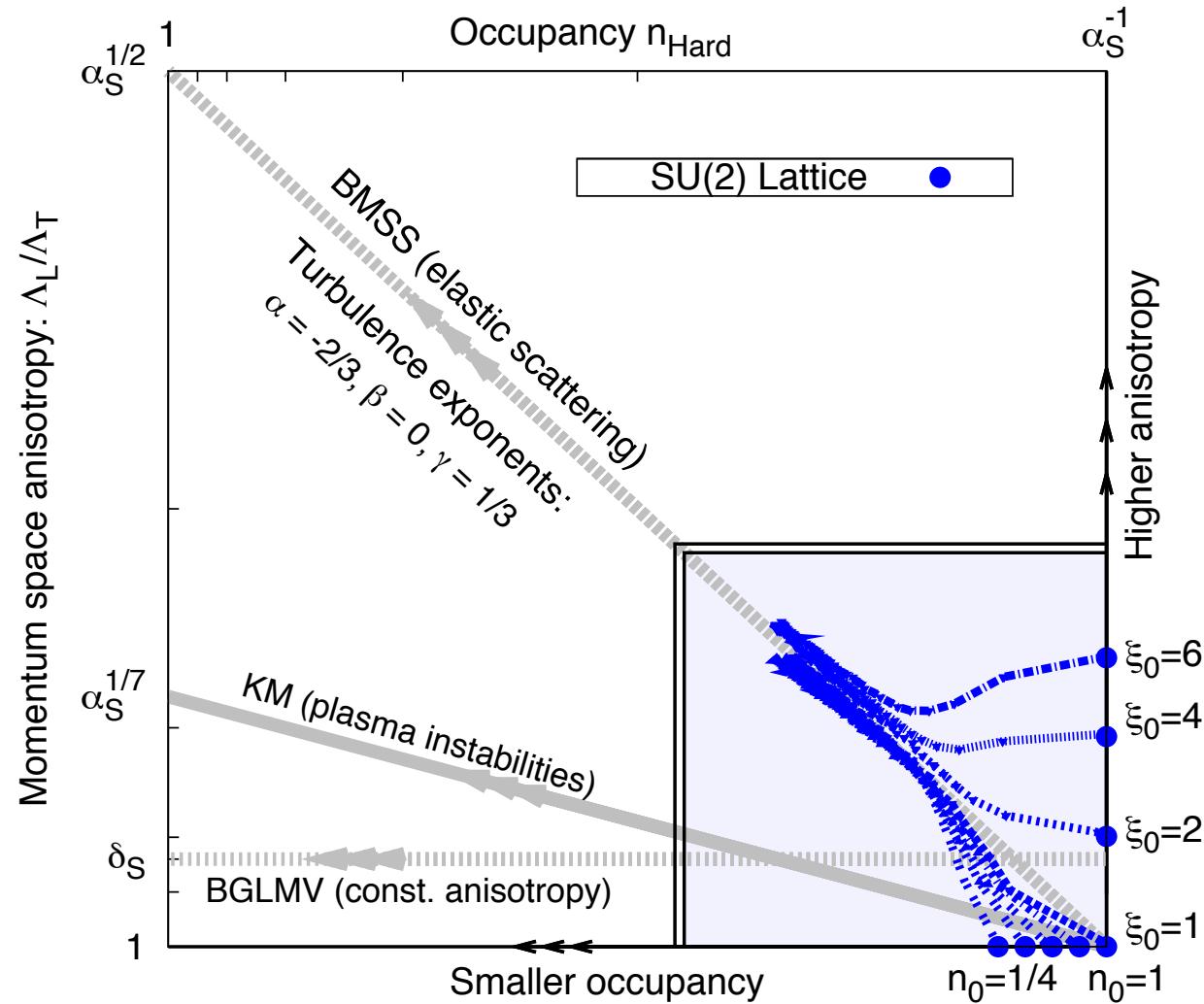
Kinetic interpretation of self-similar behavior

For self-similar scaling solution, a) small angle elastic scattering
b) energy conservation
c) number conservation

Give unique results: $\alpha = -2/3, \beta = 0, \gamma = 1/3$

- ❖ These are the same exponents (within errors) extracted from our numerical simulations !
- ❖ The same exponents appear in the “bottom-up” thermalization scenario of Baier, Mueller, Schiff, Son (BMSS)

Non-thermal fixed point in overpopulated QGP



KM: Kurkela, Moore

BGLMV: Blaizot, Gelis, Liao, McLerran, Venugopalan

Outlook

- ◆ Understand matching to ab initio Glasma initial conditions – does the same picture develop ?
- ◆ What are the implications for $f \leq 1$?
- ◆ Applicability to “Anisotropic dissipative hydrodynamics” – is thermalization necessary ? Strickland; Florkowski, Ryblewski